Linear Algebra I

Semestral Examination

Instructions: All questions carry ten marks. Vector spaces are assumed to be finite dimensional.

1. Let $\mathcal{B} = ((1,2,3),(1,0,0),(4,5,6))$ denote a basis of $\mathbb{R}^3$. Compute the coordinates of the three standard basis vectors e_1, e_2, e_3 with respect to $\mathcal{B}$.

2. Let A, B be real matrices such that the system of equations $AX = B$ has a solution in complex numbers. Then show that it also has a solution in the real numbers (i.e. all coordinates of a solution are real numbers)

3. Let L be a linearly independent subset of a vector space V over a field F. Let S be another subset of V such that $\text{Span}(S) = \text{Span}(L)$. Then prove that S contains at least as many elements as L.

4. State and prove the dimension formula for a linear transformation between two vector spaces.

5. Let V be a finite dimensional vector space over the field of real numbers. A linear operator $T : V \to V$ is called a *projection* if $T \circ T = T$. Let K and W be the kernel and image of T. Prove that $V = W + K$ and $W \cap K = \{0\}$.